قســم الاتصالات.   أسئلة الامتحان النهائي لمادة: نظرية المعلومات و الترميز

الفصل: السادس. رمز المادة: CM 303. التاريخ: 19- 07 - 2018م

كلية التقنية الالكترونية طرابلس

College of Electronic Technology - Tripoli

الفصل الدراسي: ربيع 2018   اسم الأستاذ/المنسق: حسام الدين الهنشيري.   الزمن: ساعتان.

6

الاسم: ....   رقم القيد: ....

Q1. **(10 Marks)** In a TV transmission, picture consists of 2×10^6 **elements, 32 different brightness levels** and pictures are repeated at a rate of **32 pictures per second**. I the brightness levels have **equal likelihood of occurrence** and picture elements are **independent**, find **average information rate** of this TV source?

symbol / Pic

$R_S = [sym/sec]$ $H(S) = \sum P(\,) \log_2 \frac{1}{P(\,)}$

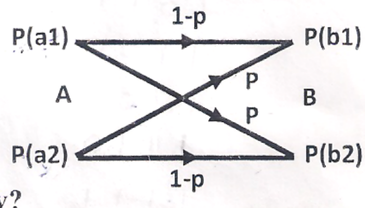
Q2. **(12 Marks, 3 each)** For the **Binary Symmetrical Channels** shown below:

a) Find the **Channel capacity** when **p = 1, p = 0, p = 0.5**, and **p = 0.3**?

b) Find the **Maximum Capacity** of the Channel?

c) Find the **Input, output, channels forward probabilities, H(A), ... H(B),** and **H(B/A)** when the channel works at **Maximum Capacity**?

d) How can you make the channel works at **half** of its **Maximum Capacity?**



Q3. **(8 Marks, 2 each)** Consider a **DMS** Source with symbols **Si, i=1,2,3,4.** Table below lists 6 possible binary codes

a) Find which of them **distinct** codes are?

b) Find which of them **prefix-free** codes are?

c) Find weather **instantaneous** codes are existence for these codes?

d) Can you decide which code is **the best code** for this source, and **why**?

Table of Codes of the source S

Si	C 1	C 2	C 3	C 4	C 5	C 6
S1	00	11	01	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

Q4. **(15 Marks, 3 each)** Consider a **Systematic Linear Block Code** whose **parity check equations** are:

$P_0 = m_0 + m_1 + m_3$, $P_1 = m_0 + m_2 + m_3$, $P_2 = m_0 + m_1 + m_2$, and $P_3 = m_1 + m_2 + m_3$,

Where m_i are the **message bits**, i = 0, 1, 2, 3, and P_i are the **check bits**, i = 0, 1, 2, 3.

(a) Find the **generator matrix** of the code and **draw the Encoder?**

(b) Find **code bits, message bits, parity bits, code rate, Hamming weight**?

(c) Find **Hamming distance,** the **error-detection** and **error-correction capabilities** of the code?

(d) Find the **syndrome look-up table?**

(e) Are the vectors [1 0 1 0 1 0 1 0] and [0 1 0 1 1 1 0 0] **valid codewords**? **(show the answer steps)**

Q5. **(4 Marks)** Given a **Binary Convolutional Encoder** with **K=3, rate 1/3**, and **Impulse Response 101011010. Encode** the input sequence bits **m = 10101**? And **write down the polynomial equations** of the encoder?

$S = eH^T$

Q6. **(3 Marks)** Consider a **(4,1,4) convolutional encoder** with the following **generator polynomials: g1 = [1010], g2 = [0101], g3 = [1110], g4 = [1001]. Draw** the encoder and how many **states** does this encoder have?

Q7. **(8 Marks, 2 each) Briefly** answer the following:

[1] How do we **measure** information content in a message?

[2] What **reduces** mutual information between input and output of a channel?

[3] What is the **purpose** of source coding and channel coding?

[4] What is the **advantages** of convolutional codes over block codes?

تمنياتي للجميع بالتوفيق،،،، أ. حسام الدين الهنشيري

لأستاذ حسام نهائي ترميز 2018/7/19

Q1 2×10^6 element/pic ≡ عدد العناصر في كل صورة

32 level عدد الالوان في كل عنصر

$\frac{1}{32}$ each level كل ~~لون~~ من الالوان ليها نفس الاحتمال

كل العناصر مستقلة

find RI → $H(s) * R_s$ عدد التكرار 32 pic/sec

$$H(S) = \sum_{i=0}^{n} P(s_i) \log_2\left(\frac{1}{P(i)}\right) =$$

$$32\left[\frac{1}{32} \log_2\left(\frac{1}{1/32}\right)\right] = \log_2(32) = 5 \text{ bit/symbol}$$

$R_s \equiv \left[\frac{\text{symbol}}{\text{sec}}\right]$ → symbol = element

$$R_s = 2 \times 10^6 \frac{\text{element}}{\text{pic}} * 32 \frac{\text{pic}}{\text{sec}} = 64 \times 10^6 \frac{\text{elem}}{\text{sec}}$$

$$R_s = 64 \times 10^6 \left[\frac{\text{symbol}}{\text{sec}}\right]$$

Q3 b) prefix-free C2 - C4 - C6

هو الكود الذي لا يكون فيه كود بداية كود اخر

Q4 Systematic linear ~~code~~ Block code

$P_0 = m_0 \oplus m_1 \oplus m_3$

$P_1 = m_0 \oplus m_2 \oplus m_3$

$P_2 = m_0 \oplus m_2 \oplus m_3$

$P_3 = m_1 \oplus m_2 \oplus m_3$

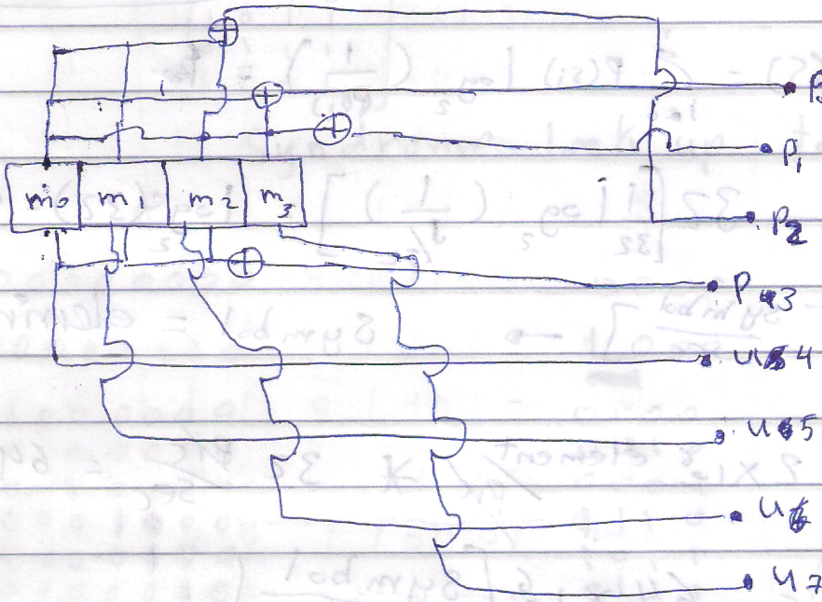
Parity bits = 4 bits

message bits = 4 bits

Q4 a)

$$P_{k*(n-k)} \quad (4*4) \qquad I_{k*k} \quad (4*4)$$

$$G = \begin{array}{c|cccc|cccc|} & p_0 & p_1 & p_2 & p_3 & & & & \\ m_0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ m_1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ m_2 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ m_3 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$



code bits = 8 "n"

message bits = 4 k

parity bits = 4 P

code rate = $k/n = 4/8 = 1/2$

Hamming weight = عدد الواحدات في codewords

message	codeword	Hmming weight
0000	00000000	[illegible]
0001	11010001	4
0010	01110010	4
0011	10100011	4
0100	10110100	4
0101	01100101	4
0110	11000110	4
0111	00010111	4
1000	11101000	4
1001	00111001	4
1010	10011010	4
1011	01001011	4
1100	01011100	4
1101	10001101	4
1110	00101110	4
1111	11111111	8

Hamming distance ≡ هي جمع كل codeword مع الاخرين بطريقة XOR

d_{min} = Hmming weight mimum = 4

$e = d_{min} - 1 = 4 - 1 = 3$ bit

$t = \left\lfloor \frac{e}{2} \right\rfloor = \left\lfloor \frac{3}{2} \right\rfloor = \lfloor 1.5 \rfloor = 1$ bit

d) $H = \left[\begin{array}{cccc|cccc} 1&0&0&0&1&1&0&1 \\ 0&1&0&0&1&0&1&1 \\ 0&0&1&0&1&1&1&0 \\ 0&0&0&1&0&1&1&1 \end{array}\right]$ (I_{n-k} | P^T)

$$H^T = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&1&1&0 \\ 1&0&1&1 \\ 0&1&1&1 \\ 1&1&0&1 \end{bmatrix}$$

Syndrome look up table

e	S
000000000	0000
100000000	1000
01000000	0100
00100000	0010
00010000	0001
00001000	1110
00000100	1011
00000010	0111
00000001	1101

$V_1 = [10101010]$ $V_2 = [01011100]$

لو ضربنا V_1 في H^T و كان ناتج الضرب 0000 اذا V_1 موجود Valid
نفس الشيء بالنسبة لـ V_2

$$V_1[10101010] \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&1&1&0 \\ 1&0&1&1 \\ 0&1&1&1 \\ 1&1&0&1 \end{bmatrix} = 0101 \quad \text{Not valid}$$

$$V_2[01011100] \cdot H^T = 0000 \quad \text{Valid}$$

2018 / 7 / 19

5

Binary Convolutional Encoder

Q5

$K = 3$ $\quad$ $rate = \frac{\text{input bit}}{\text{out put bit}} = \frac{1}{3}$

impulse Response = 101011010

m = 10101

نكتب المسج متاعنا بشكل عمودي وكل ما نلقى 1 نكتبوا impulse Response و لما نلقى "صفر" 0 نكتبوا صفار بعدد bits الامبلس وكل ما ننزل سطر نديروا ازاحة بعدد output bit ومن ثم نجمعوا XOR

m

1 $\quad$ 1 0 1 0 1 1 0 1 0 $\quad \oplus$

ازاحة 3 bit →

0 $\quad$ 0 0 0 0 0 0 0 0 0 $\quad \oplus$

1 $\quad$ 1 0 1 0 1 1 0 1 0 $\quad \oplus$

0 $\quad$ 0 0 0 0 0 0 0 0 0 $\quad \oplus$

1 $\quad$ 1 0 1 0 1 1 0 1 0

U = 1 0 1 0 1 1 1 1 1 0 1 1 1 1 0 0 1 1 0 1 0 $\quad$ هذا codeword

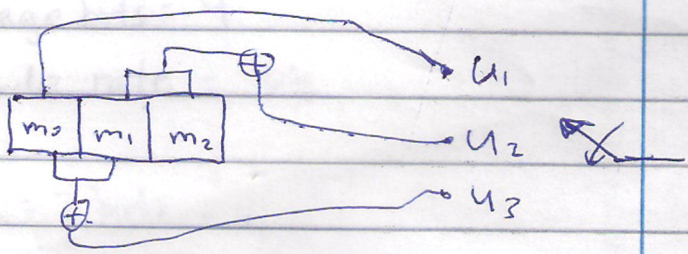
(b)

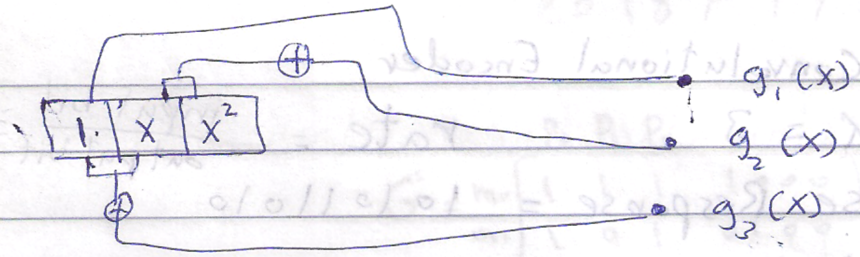
		u_1	u_2	u_3
[1 | 0 | 0]	m_0	1	0	1
[0 | 1 | 0]	m_1	0	1	1
[0 | 0 | 1]	m_2	0	1	0

$u_1 = m_0$

$u_2 = m_1 \oplus m_2$

$u_3 = m_0 \oplus m_1$

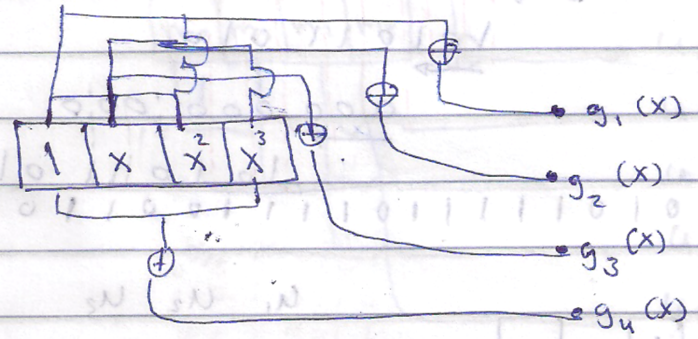




$g_1(x) = 1$ ، $g_2(x) = x \oplus x^2$ ،
$g_3(x) = 1 \oplus x$

Q6 $K = 4$ ، $k = 1$ input ، $n = 4$ output

$g_1(x) = 1010$ ، $g_2(x) = 0101$ ، $g_3(x) = 1110$
$g_4(x) = 1001$



$g_1(x) = 1 \oplus x^2$
$g_2(x) = x \oplus x^3$
$g_3(x) = 1 \oplus x \oplus x^2$
$g_4(x) = 1 \oplus x^3$

$8 = 2^3 = 2^{K-1} =$ عدد الحالات
bit $3 = K-1 =$ عدد خانات الحالات

ية التقنية الالكترونية
College of Electronic Technology - Tripoli

قسم الاتصالات. أسئلة الامتحان النهائي لمادة: نظرية المعلومات و الترميز

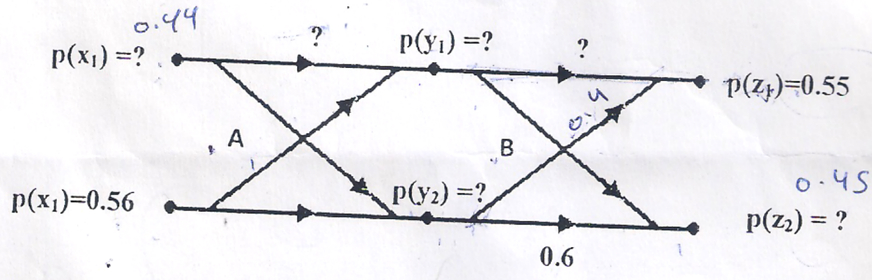
الفصل: السادس. رمز المادة: CM 303. التاريخ: 31 - 01 - 2018م

الفصل الدراسي: خريف 2017 اسم الأستاذ/المنسق: حسام الدين الهنشيري. الزمن: ساعتان.

الاسم: .... رقم القيد: ....

5

Q1. (12 Marks) For the two **Binary Symmetrical Channels** (A & B) connected in cascade as shown below, Find the **input, output and channels forward probabilities.**



Q2. (8 Marks) Consider a DMS Source with symbols Si, i=1,2,3,4. Table below lists 6 possible binary codes

Table of Codes of the source S

Si	Code 1	Code 2	Code 3	Code 4	Code 5	Code 6
S1	00	11	0	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

$Y_1 = 0.44$

a) Find which of them **distinct** codes are? C2 C3 C4 C5 C6 (2 Marks)

b) Find which of them **prefix-free** codes are? C2 C4 C6 (2 Marks)

c) Find weather **instantaneous** codes are existence for these codes? (2 Marks)

d) Can you decide which code is the best code for this source, and why? (2 Marks)

Q3. (24 Marks, 3 each) Consider the Systematic Linear Block Code with the following syndrome look-up table.

Error Pattern (e)	Syndrome (S)
0000000	000
0000001	110
0000010	011
0000100	101
0001000	111
0010000	001
0100000	010
1000000	100

a) Find the **Generator matrix** of the code? أقل عدد واحدات I 100 010 001

b) Find all the **Code Words** and the **Minimum Hamming Distance**?

c) Find **code bits, message bits, parity bits, code rate,** the **error-detection** and **error-correction capabilities** of the code?

d) Write down **the Parity Check Equations** and **draw the Encoder**? 11

e) Are the generator vectors **linearly independent**? (Justify your answer) 101

f) Is this code a **linear code**? (Justify your answer) 011

g) Encode the bit stream, m= 11011110001001...? 110

h) If r_1 = 1101011 and r_2= 0101101 were received, what are the transmitted code-words and original messages? (*clearly show the recovery steps*)

$r = \frac{k}{n}$ $S = eH^T$

Q4. (16 Marks) Given a Binary Convolutional Encoder with K=3, rate 1/3, and Impulse Response 101011010.

a) Encode the input sequence bits m = 10101? (5 Marks)

b) Draw the encoder? (show your answer) (6 Marks)

c) Draw the Trills diagram of the encoder? (5 Marks)

$v_1 v_2 v_3$ 101 011 010

101 e^{-1} $S = H^T$ كيف !!

100 101
010 011
001 011

1 1 0 1 0 0 0 0
1 0 1 1 0 1 0 0
1 1 0 0 0 0 1 0

تمنياتي للجميع بالتوفيق
حسام الدين الهنشيري

$S = eH^T$ P^T $S = rH^T$ $I_{k \times k}$ $P_{n-k \times k}$

ترميز نهائي لأستاذ [illegible] 2018/1/31

7

Q3 Systamatic linear Block ~~diagram~~ code

$$H^T = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \\ \hline 1&1&1 \\ 1&0&1 \\ 0&1&1 \\ 1&1&0 \end{bmatrix} \begin{matrix} I_{n-k} \\ {}_{n-k} \\ \\ P \\ {}_{k*(n-k)} \end{matrix}$$

$$G = \begin{bmatrix} 1&1&1&1&0&0&0 \\ 1&0&1&0&1&0&0 \\ 0&1&1&0&0&1&0 \\ 1&1&0&0&0&0&1 \end{bmatrix}$$

m	
0 0 0 0	0 0 0 0 0 0 0
0 0 0 1	1 1 0 0 0 0 1
0 0 1 0	0 1 1 0 0 1 0
0 0 1 1	1 0 1 0 0 1 1
0 1 0 0	1 0 1 0 1 0 0
0 1 0 1	0 1 1 0 1 0 1
0 1 1 0	1 1 0 0 1 1 0
0 1 1 1	0 0 0 0 1 1 1
1 0 0 0	1 1 1 1 0 0 0
1 0 0 1	0 0 1 1 0 0 1
1 0 1 0	1 0 0 1 0 1 0
1 0 1 1	0 1 0 1 0 1 1
1 1 0 0	0 1 0 1 1 0 0
1 1 0 1	1 0 0 1 1 0 1
1 1 1 0	0 0 1 1 1 1 0
1 1 1 1	1 1 1 1 1 1 1

$d_{min} = 3$

code bits = 7   message bits = 4

Parity bits = 3   code rate = 4/7

$e = d_{min} - 1 = 3 - 1 = 2$ bits

$t = \lfloor \frac{e}{2} \rfloor = \frac{2}{2} = 1$ bits

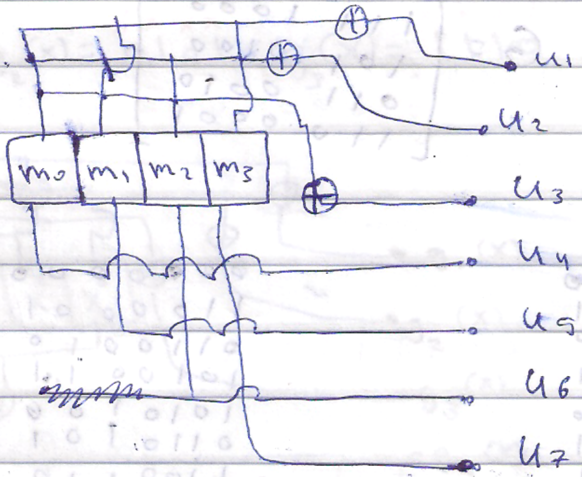
Q3 d)

$$G = \begin{array}{c|ccc|cccc} & P_0 & P_1 & P_2 & & & & \\ m_0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ m_1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ m_2 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ m_3 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \end{array}$$

$P_0 = m_0 \oplus m_1 \oplus m_3$

$P_1 = m_0 \oplus m_2 \oplus m_3$

$P_2 = m_0 \oplus m_1 \oplus m_2$



e)

لكي نعرف generator vector هو مستقل خطيا ام لا

نجمع أي 2 من الصفوف في G matrix لو الناتج كان

صف غير موجود في G matrix اذا هو مستقل خطيا

صف الاول من G $\quad$ 1 1 1 1 0 0 0

الثاني // في G $\quad \oplus$ 1 0 1 0 1 0 0

غير موجود في G $\quad$ 0 1 0 1 1 0 0

اذا مستقل خطيا

f)

اذا جمعنا أي 2 من codewords بـ XOR و الناتج كان codeword

موجود في الجدول اذا ~~الخطأ~~ الكود خطي

Q3) g

$m = 1101\ 1110\ 0010\ 01 \ldots\ldots$

$\underbrace{1001101}\ \underbrace{0011110}\ \underbrace{0110010}$

(h)

$r_1 = 1101011 \rightarrow *$

$r_2 = 0101101 \rightarrow *$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\Rightarrow S_1 = 100$

$\Rightarrow S_2 = 110$

$S = r H^T$

$U = r \oplus e$

$\therefore e_1 = 1000000$

$e_2 = 0000001$

$$U_1 = \begin{array}{r} r_1\ 1101011 \\ \oplus\ e_1\ 1000000 \\ \hline 010\underbrace{1011}_{m_1} \end{array}$$

$$U_2 = \begin{array}{r} r_2\ 0101101 \\ \oplus\ e_2\ 0000001 \\ \hline 010\underbrace{1100}_{m_2} \end{array}$$

أخر 4 bit هي message

$m_1 = 1011$

$m_2 = 1100$

Q4) $K = 4$ rate $= \frac{1}{3}$ $\frac{\text{input}}{\text{output}}$

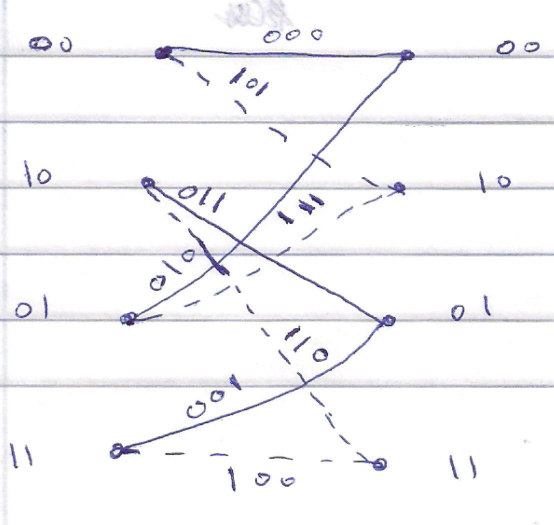
Impulse Response $= 101011010$

$m = 10101$

2018/7/19 الفقرتين a,b موجودين في أسئلة

السؤال 5

(c)



كلية التقنية الالكترونية

قســم الاتصالات.   اسئلة الامتحان النهائي لمادة: نظرية المعلومات و الترميز

الفصل: السادس كلية/الاول استدراكي. رمز المادة: CM 303 . التاريخ: 21-01-2017

الفصل الدراسي: خريف 2016   اسم الأستاذ/المنسق: حسام الدين الهشيري.   الزمن: ساعتان.

Q1. A discrete source transmits messages x_1, x_2, x_3 with probabilities $p(x_1)= 0.3$, $p(x_2)= 0.25$, $p(x_3)= 0.45$. The source is connected to the channel whose conditional probability matrix is

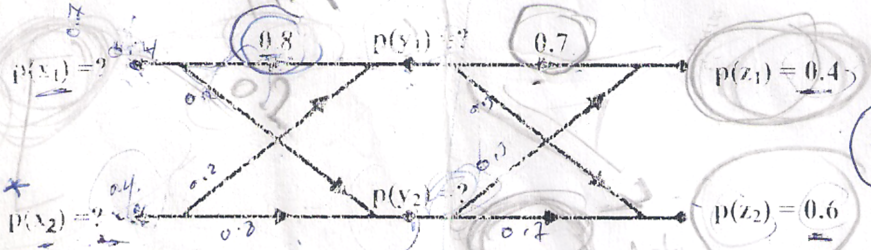
$$P(Y/X) = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

$[0.3 \;\; 0.25 \;\; 0.45]_{1\times3}$ 3×3 $= p(b_j/a_i)\,p(a_i)$

A. Draw the channel schematic? And Obtain the joint probability matrix $P(X, Y)$? (4 Marks)

B. Obtain the probabilities $p(y_1)$, $p(y_2)$ and $p(y_3)$? (6 Marks)

Q2. Consider two binary symmetrical channels are connected in cascade as shown below:



أكثر جدول أعمال = 1
لما تكون امتحان كبير نشوف كمية المعلومات أقل

A. Find the probabilities $p(x_1)$, $p(x_2)$, $p(y_1)$ and $p(y_2)$? (8 Marks)

Q3. (16 Marks) Consider a source with a six symbol alphabet, X_1, X_2, X_3, X_4, X_5, and X_6, with probabilities $P_1 = 0.2$, $P_2 = 0.01$, $P_3 = 0.35$, $P_4 = 0.38$, $P_5 = 0.02$, and $P_6 = 0.04$, respectively.

A. Find a Shannon-Fano code for this source. (8 Marks) $t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = 1$

B. Compute the average code length of the code. (8 Marks)

Q4. (10 Marks) Consider the Systematic Linear Block Code with the following parity check matrix.

$u = mG$

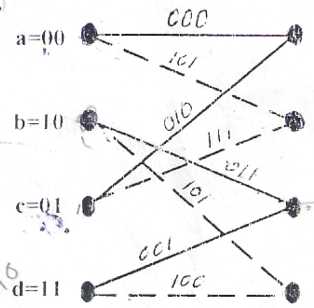
$$H = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

A. If the code is a single error correction code, find the syndrome look-up table. (5 Marks)

B. For the received vectors r_1 and r_2, recover the transmitted code-word and the original message.

$r_1 = 1101011$, $r_2= 0101101$. (*clearly show the recovery steps*) (6 Marks)

Q5. (15 Marks) Given the following trellis diagram of a convolutional code.



A. Encode the input sequence bits $m = 10101$ 100

B. Decode the received vector r using Viterbi Decoding.

R= 101 111 001 000 000.

C. How many errors are there in the received vector R (identify the error bits) and show the transmitted vector U

e $S = eH^T$

تمنياتي للجميع بالتوفيق

استاذ المادة: أ. حسام الدين الهشيري

2017/1/21

Q1 $P(x_1) = 0.3$, $P(x_2) = 0.25$, $P(x_3) = 0.45$

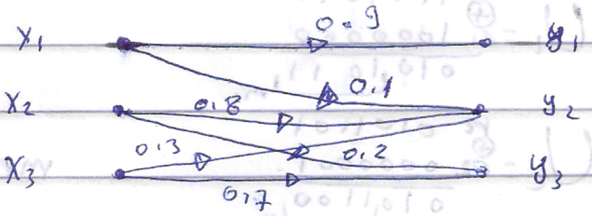
$$P(y/x) = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.9 & 0.1 & 0 \\ x_2 & 0 & 0.8 & 0.2 \\ x_3 & 0 & 0.3 & 0.7 \end{array}$$

$P(y_1/x_1) = 0.9$, $P(y_2/x_1) = 0.1$, $P(y_3/x_1) = 0$

$P(y_1/x_2) = 0$, $P(y_2/x_2) = 0.8$, $P(y_3/x_2) = 0.2$

$P(y_1/x_3) = 0$, $P(y_2/x_2) = 0.3$, $P(y_3/x_3) = 0.7$

$P(X, Y) \rightarrow$ joint



$y_1 = P(x_1)\, P(y_1/x_1) = 0.3 * 0.9 = 0.27$

$y_2 = P(x_1) * P(y_2/x_1) + P(x_2) P(y_2/x_2) + P(x_3) P(y_2/x_3)$

$= (0.3 * 0.1) + (0.25 * 0.8) + (0.45 * 0.3) = 0.365$

$y_3 = P(x_2) P(y_3/x_2) + P(x_3) P(y_3/x_3) =$

$= (0.25 * 0.2) + (0.45 * 0.7) = 0.365$

$P(X, Y) = \dfrac{P(x_i/y_j)}{P(y_j)} =$

$P(x_1, y_1) =$

$P(x_1, y_2) =$

$P(x_2, y_1) =$

$P(x_2, y_2) =$